WHAT THE **** IS A STACK AND WHY SHOULD I CARE?

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Abstract. Schemes are the basic object of study in algebraic geometry, which are usually defined as ringed spaces. We introduce an alternative formalism, the so-called *functor-of-points* perspective on schemes. This motivates the definition of stacks, which are certain functors on rings. The main insight of Grothendieck (please correct my history!) was that one can really think of such functors as geometric objects, and develop the theory of sheaves on them.

1. Functor-of-points

The basic object of study in modern algebraic geometry is a scheme. Usually, schemes are defined as (locally) ringed spaces $(X, \mathcal{O}_X)$ which locally looks like $(\operatorname{Spec} A, \mathcal{O}_{\operatorname{Spec} A})$.

An alternative way to think about schemes is via their *functor-of-points*.

Definition 1.1. For a scheme X, let the *functor-of-points* be:

$$F_X \colon \mathrm{CRings} = \mathrm{AffSch}^{\mathrm{op}} \to \mathrm{Sets}$$
$$R \mapsto X(R) := \operatorname{Hom}_{\mathrm{Sch}}(\operatorname{Spec}(R), X).$$

Example 1.2. The following are examples of functors-of-points of schemes:

1. when $X = \mathbb{A}^1_{\mathbb{Z}} = \operatorname{Spec} \mathbb{Z}[t]$,
$$X(R) = R.$$
2. when $X = \mathbb{G}_{m,\mathbb{Z}} = \operatorname{Spec} \mathbb{Z}[t^{\pm 1}]$,
$$X(R) = R^{\times} = \{x \in R : \text{invertible}\}.$$
3. the circle: $X = \operatorname{Spec}(\mathbb{Z}[x, y]/(x^2 + y^2 - 1))$, then
$$X(R) = \{(x, y) \in R^2 : x^2 + y^2 = 1\}.$$
More generally, for $X = \operatorname{Spec}(\mathbb{Z}[t_1, \ldots, t_n]/(f_1, \ldots, f_m))$, we have
$$X(R) = \{\underline{x} = (x_1, \ldots, x_n) \in R^n : f_1(\underline{x}) = \cdots = f_m(\underline{x}) = 0\}.$$
Thus, F_X really just captures all the solutions to the equations $f_1, \ldots, f_m$!
4. (open complements) consider the open subscheme $U = \mathbb{A}^2_{\mathbb{Z}} \backslash \{(0, 0)\}$ of $X = \mathbb{A}^2_{\mathbb{Z}}$. Then
$$X(R) = R^2$$
and U defines the subset
$$U(R) = \{(x, y) \in R^2 : x \text{ and } y \text{ generate the unit ideal}\}.$$
More generally, for $U = \mathbb{A}^n_{\mathbb{Z}} \backslash \{(0, \ldots, 0)\}$, we have
$$U(R) = \{(x_1, \ldots, x_n) \in R^n : x_1, \ldots, x_n \text{ generate the unit ideal}\},$$
generalizing (2.). More generally, the open complement U of $Z = V(f_1, \ldots, f_m)$ in $X = \mathbb{A}^n_{\mathbb{Z}}$ has functor-of-points
$$U(R) = \{\underline{x} \in R^n : f_1(\underline{x}), \ldots, f_m(\underline{x}) \text{ generate the unit ideal}\}.$$

Even more generally, given a scheme $X = \mathrm{Spec}(A)$ and a closed subscheme $Z = V(I)$, the open complement $U = X \backslash Z$ has functor-of-points

$$X(R) = \{\varphi\colon A \to R : \varphi(I)R = R\}.$$

5. (projective line) for a further non-affine example, consider $X = \mathbb{P}^1_{\mathbb{Z}}$. Then

$$X(R) = \{\text{invertible submodules } L \subset R^2 \text{ s.t. } R^2/L \text{ is also invertible}\},$$

where a R-module L is *invertible* if it is locally free of rank one. More generally, when $X = \mathbb{P}^n_{\mathbb{Z}}$, then

$$X(R) = \{\text{invertible submodules } L \subset R^{n+1} \text{ s.t. } R^{n+1}/L \text{ is locally free}\}.$$

This is the "moduli description" of $\mathbb{P}^n$.

More generally, let:

Definition 1.3. A *prestack* is a functor CRings $\to$ Sets. Denote PreStk := Fun(CRings, Sets).

Prestacks can really be thought of as generalizations of schemes:

Theorem 1.4. *The functor $X \mapsto F_X$ defines a fully faithful embedding* Sch $\hookrightarrow$ PreStk.

Proof. We need to prove that for any schemes X and Y the natural map

$$\mathrm{Hom}_{\mathrm{Sch}}(Y, X) \to \mathrm{Nat}(F_Y, F_X)$$

is a bijection. When Y is affine this is Yoneda. Generally, cover Y by open affines $\{U_i\}_{i\in I}$. The key observation is that

$$\mathrm{Hom}_{\mathrm{Sch}}(Y, X) = \{(\varphi_i) \in \prod_{i\in I} \mathrm{Hom}_{\mathrm{Sch}}(U_i, X) : \varphi_i|_{U_i\cap U_j} = \varphi_j|_{U_i\cap U_j}\}. \tag{1.1}$$

□

Remark 1.5. In fact, if you want you can define a scheme to be a prestack which can locally be covered by affine schemes.

The equality (1.1) expresses the idea that a morphism $X \to Y$ can be recovered from local data. If we want to think of a functor $F\colon$ CRings $\to$ Sets as a "geometric" object, we certainly want that property. We define a *stack* to be a prestack which can be recovered from local data in this sense.

Definition 1.6. A prestack $F\colon$ CRings $\to$ Sets is a *stack* if for any distinguished open cover

$$\mathrm{Spec}(R) = \bigcup_{i\in I} \mathrm{Spec}(R_{f_i}),$$

we have

$$F(R) = \{x_i \in \prod_{i\in I} F(R_{f_i}) : x_i|_{R_{f_i f_j}} = x_j|_{R_{f_i f_j}} \in F(R_{f_i f_j})\}.$$

Since schemes are covered by affine opens, stacks uniquely extend to a functor $\mathrm{Sch}^{\mathrm{op}} \to$ Sets, which we will also denote as F by an abuse of notation, such that for any open cover $Y = \bigcup U_i$,

$$F(Y) = \{(x_i) \in \prod_{i\in I} F(U_i) : x_i|_{U_i\cap U_j} = x_j|_{U_i\cap U_j}\}.$$

Remark 1.7. More generally, given a Grothendieck topology on Sch, one can ask for a prestack to satisfy the sheaf axiom with respect to the topology. Given a cover $\{U_i\}$ of Y, we require

$$F(Y) = \{(x_i) \in \prod_{i\in I} F(U_i) : x_i|_{U_i\times_X U_j} = x_j|_{U_i\times_X U_j}\}.$$

Example 1.8. Formal schemes, as introduced by Dhruv last time, are an examples of prestacks which are not stacks. Let R be a noetherian ring, and let I be an ideal. Then $X = \mathrm{Spf}_I(R)$ can be considered a stack by

$$X(S) = \lim_n \mathrm{Hom}(R/I^n, S).$$

More generally, let X be a noetherian scheme and let $\mathcal{I}$ be a sheaf of ideals cutting out a closed subscheme $Y \hookrightarrow X$. Let $Y_n = \underline{\mathrm{Spec}}(\mathcal{O}_X/\mathcal{I}^n)$ be the n-th formal neighborhood of Y in X. Then $X^\wedge_Y$ is a stack by

$$(X^\wedge_Y)(S) = \lim_n \mathrm{Hom}_{\mathrm{Sch}}(S, Y_n).$$

2. Examples of stacks

We will have many more interesting examples if we extend Defintion 1.3 slightly to:

Definition 2.1. A *prestack* is a functor CRings $\to$ Gpds, where Gpds is the category of groupoids (=categories whose morphisms are invertible). Denote $\mathrm{PreStk} := \mathrm{Fun}(\mathrm{CRings}, \mathrm{Gpds})$.

Example 2.2. The following are some examples of stacks valued in groupoids:

1. Any scheme X is a prestack, by setting $F_X(R) = X(R)$ which is considered a groupoid with objects $X(R)$ and the only morphisms are the identities.
2. (classifying stacks) The stack $\mathfrak{X} = B\mathbb{G}_m$ is defined by:

$$\mathrm{ob}\,\mathfrak{X}(R) = \{\text{line bundle on } \mathrm{Spec}\, R\}$$
$$\mathrm{Hom}_{\mathfrak{X}(R)}(\mathcal{L}, \mathcal{L}') = \{\text{isomorphism } \varphi\colon \mathcal{L} \to \mathcal{L}'\},$$

and is called the *classifying stack of line bundles.* More generally, for an group scheme G, a *G-bundle* on a scheme X is a scheme $p\colon Y \to X$ with a G-action on Y commuting with p such that there exists an open cover U_i of X with $p^{-1}(U_i) \cong G \times U_i$. Then $\mathfrak{X} = BG$ is defined by:

$$\mathrm{ob}\,\mathfrak{X}(R) = \{G\text{-bundle } Y \text{ on } \mathrm{Spec}\, R\}$$
$$\mathrm{Hom}_{\mathfrak{X}(R)}(Y, Y') = \{G\text{-equivariant isomorphism } \varphi\colon Y \to Y' \text{ over } X\}.$$

When $G = \mathbb{G}_m$, this recovers the previous desciption of $B\mathbb{G}_m$, because given a line bundle $\mathcal{L}$ on $\mathrm{Spec}(R)$, we may associate a $\mathbb{G}_m$-bundle $Y = \mathrm{Isom}(\mathcal{O}, \mathcal{L})$ defined by:

$$Y(S) = \{\text{a morphism } \mathrm{Spec}\, S \xrightarrow{\varphi} \mathrm{Spec}\, R, \text{ with an isomorphism } \mathcal{O}_{\mathrm{Spec}\, S} \xrightarrow{\iota} \varphi^*\mathcal{L}\}.$$

Here $\mathbb{G}_m$ acts on Y by changing the trivialization, i.e., $a \in \mathbb{G}_m(S) = S^\times$ acts by $(\varphi, \iota) \mapsto (\varphi, a\iota)$. Conversely, given a $\mathbb{G}_m$-bundle Y, we the product $Y \times^{\mathbb{G}_m} \mathbb{A}^1 := (Y \times \mathbb{A}^1)/\mathbb{G}_m$ is the total space of a line bundle $\mathcal{L}$. This gives an equivalence

$$\{\text{line bundles } \mathcal{L} \text{ on } \mathrm{Spec}\, R\} \simeq \{\mathbb{G}_m\text{-bundle } Y \text{ on } \mathrm{Spec}\, R\}$$
$$\mathcal{L} \mapsto \mathrm{Isom}(\mathcal{O}, \mathcal{L})$$
$$Y \times^{\mathbb{G}_m} \mathbb{A}^1 \leftarrow Y.$$

As further examples, $B\mathrm{GL}_n$ classifies rank n vector bundles, $B\mathrm{SL}_n$ classifies rank n vector bundles with a trivialization of the determinant, and $B\mathrm{Sp}_{2n}$ classifies rank $2n$ vector bundles with a non-degenerate symplectic form.
3. (quotient stacks) For a scheme X with an action of an group scheme G, the quotient stack $\mathfrak{X} = [X/G]$ is defined by:

$$\mathrm{ob}\,\mathfrak{X}(R) = \{G\text{-bundle } Y \xrightarrow{p} \mathrm{Spec}\, R \text{ and a } G\text{-equivariant morphism } Y \to X\}$$
$$\mathrm{Hom}_{\mathfrak{X}(R)}((Y, p), (Y', p')) = \{\text{isomorphism } \varphi\colon Y \to Y' \text{ such that } p' \circ \varphi = p\}.$$

For example, $\mathbb{G}_m$ acts on $\mathbb{A}^1$ by scaling. The quotient stack $\mathfrak{X} = [\mathbb{A}^1/\mathbb{G}_m]$ is defined by:

$$\begin{aligned} \operatorname{ob}\mathfrak{X}(R) &= \{\text{line bundle } \mathcal{L} \text{ on } \operatorname{Spec}(R), \text{ and a homomorphism } s\colon \mathcal{L} \to \mathcal{O}\} \\ \operatorname{Hom}_{\mathfrak{X}(R)}((\mathcal{L}, s), (\mathcal{L}', s')) &= \{\text{an isomorphism } \varphi\colon \mathcal{L} \to \mathcal{L}' \text{ such that } s' \circ \varphi = s\}. \end{aligned}$$

Indeed, a R-point of $\mathbb{A}^1/\mathbb{G}_m$ is the data of a line bundle $\mathcal{L}$ together with a $\mathbb{G}_m$-equivariant morphism $\operatorname{Isom}(\mathcal{O}, \mathcal{L}) \to \mathbb{A}^1$. Any such homomorphism arises as post-composition with a homomorphism $s\colon \mathcal{L} \to \mathcal{O}$. The stack $[\mathbb{A}^1/\mathbb{G}_m]$ has a closed substack $[0/\mathbb{G}_m]$ consisting of those pairs $(\mathcal{L}, s)$ where $s = 0$, and the open substack $[\mathbb{G}_m/\mathbb{G}_m] = \mathrm{pt}$ consisting of those pairs $(\mathcal{L}, s)$ where s is an isomorphism.[1]

4. A classical motivation for stacks is to consider the space $\mathcal{M}_g$ of genus g curves. For a scheme X, the groupoid $\mathcal{M}_g(X)$ consists of relative curves $C \to X$ whose fibers are genus g curves. This is genuinely a groupoid. For example, the Klein quartic, cut out by $x^3y + y^3z + z^3x = 0$ in $\mathbb{P}^2$, is a genus 3 curve with automorphism group of order 168. An easier example is that any hyperelliptic curve has an order two automorphism!

For prestacks valued in groupoids, we also want to discuss the notion "recovering from local data," analogous to Definition 1.6. Naively, we would require that for an open cover $\{U_i\}$ of Y,

$$F(Y) = \{(x_i) \in \prod_{i\in I} F(U_i) : x_i|_{U_i \cap U_j} = x_j|_{U_i\cap U_j}\}.$$

However, as our experience in category theory will tell us, equality is not the correct notion, since it is not preserved under category equivalences. Thus, our next approximation would be to require:

$$F(Y) = \{(x_i) \in \prod_{i\in I} F(U_i) : x_i|_{U_i \cap U_j} \simeq x_j|_{U_i\cap U_j}\}$$

where this is an isomorphism in the groupoid $F(U_i \cap U_j)$. However, our experience in category theory also tells us that requiring an isomorphism is also not the correct notion. Rather, we should remember *which isomorphisms* exist between $x_i|_{U_i\cap U_j}$ and $x_j|_{U_i\cap U_j}$. Thus, we arrive at the following definition:

Definition 2.3. A prestack $F\colon$ CRings $\to$ Sets is a *stack* if for any affine scheme $Y = \operatorname{Spec}(R)$ and a distinguished open cover $\{U_i = \operatorname{Spec}(R_{f_i})\}_{i\in I}$, the groupoid $F(Y)$ is equivalent to the groupoid whose objects are

- objects $x_i \in F(U_i)$ for any $i \in I$;
- isomorphisms $\alpha_{ij}\colon x_i|_{U_i\cap U_j} \simeq x_j|_{U_i\cap U_j}$ for each $i, j \in I$ such that:
  - $\alpha_{ii} = \mathrm{id}_{x_i}$ for any $i \in I$;
  - $\alpha_{ij} = \alpha_{ji}^{-1}$ for any $i, j \in I$; and
  - for any $i, j, k \in I$, the following diagram commutes:

$$\begin{tikzcd} x_i|_{U_{ijk}} \arrow[rr, "\alpha_{ik}|_{U_{ijk}}"] \arrow[dr, "\alpha_{ij}|_{U_{ijk}}"'] & & x_k|_{U_{idk}} \\ & x_j|_{U_{ijk}}, \arrow[ur, "\alpha_{jk}|_{U_{ijk}}"'] & \end{tikzcd}$$

[1]Though we didn't define open and closed substacks, there are perfectly well-behaved such notions.

and whose morphisms are isomorphisms $\gamma_i \colon x_i \simeq x_i'$ such that the following diagram commutes for any $i, j \in I$:

$$\begin{tikzcd} x_i|_{U_{ij}} \arrow[r, "\gamma_i|_{U_{ij}}"] \arrow[d, "\alpha_{ij}"] & x_i'|_{U_{ij}} \arrow[d, "\alpha_{ij}'"] \\ x_j|_{U_{ij}} \arrow[r, "\gamma_j|_{U_{ij}}"] & x_j'|_{U_{ij}}. \end{tikzcd}$$

3. Representability

We can define the fiber product of stacks in a way that generalizes the fiber product of schemes. For this, let $f \colon X \to S$ and $g \colon Y \to S$ be morphisms of schemes. Then by the universal property of fiber products, a morphism $\mathrm{Spec}(R) \to X \times_S Y$ is equivalent to morphisms $\alpha \colon \mathrm{Spec}(R) \to X$ and $\beta \colon \mathrm{Spec}(R) \to Y$ such that $f \circ \alpha = g \circ \beta$. Thus, we can define

Definition 3.1. Let $\mathfrak{X}_1$, $\mathfrak{X}_2$, and $\mathfrak{T}$ be stacks with morphisms $f \colon \mathfrak{X}_1 \to \mathfrak{T}$ and $g \colon \mathfrak{X}_2 \to \mathfrak{T}$. Then for a ring R, we have:

$$(\mathfrak{X}_1 \times_{\mathfrak{T}} \mathfrak{X}_2)(R) = \{x_i \in \mathfrak{X}_i(R) : \iota \colon f^* x_1 \simeq g^* x_2\}.$$

A useful notion in the theory of stacks is representability. The idea is that even if the stacks $\mathfrak{X}$ and $\mathfrak{Y}$ are not schemes, the morphism $\mathfrak{X} \to \mathfrak{Y}$ may look like a morphism of schemes.

Definition 3.2. A morphism of stacks $\mathfrak{X} \to \mathfrak{Y}$ is *representable* if for any ring R and a morphism $\mathrm{Spec}(R) \to \mathfrak{Y}$, the fiber product $\mathfrak{X} \times_{\mathfrak{Y}} \mathrm{Spec}(R)$ is an affine scheme. Analogously, a morphism of stacks $\mathfrak{X} \to \mathfrak{Y}$ is *schematic* if for any scheme X and a morphism $X \to \mathfrak{Y}$, the fiber product $\mathfrak{X} \times_{\mathfrak{Y}} X$ is a scheme.

Many statements about representable (resp., schematic) morphisms of stacks can be formally reduced to the analogous statement about affine schemes (resp., schemes). Moreover, many morphisms of stacks we encounter in the wild are representable!

Example 3.3. Let G be a group scheme. Then the morphism $\mathrm{pt} \to BG$ is schematic. Indeed, for any morphism $X \to BG$, which corresponds to a G-bundle $Y \to X$, we have

$$X \times_{BG} \mathrm{pt} \simeq Y.$$

Indeed, a R-point of $X \times_{BG} \mathrm{pt}$ amounts to a point $f \colon \mathrm{Spec}(R) \to X$ and a point $g \colon \mathrm{Spec}(R) \to \mathrm{pt}$ (this is no data) with an identification of the post-compositions with $X \to BG$. The composition $\mathrm{Spec}(R) \to X \to BG$ corresponds to the G-bundle $\mathrm{Spec}(R) \times_X Y$ on $\mathrm{Spec}(R)$. Thus, the data of a R-point of $X \times_{BG} \mathrm{pt}$ amounts to a trivialization of the G-bundle $\mathrm{Spec}(R) \times_X Y$, i.e., a section of the map $\mathrm{Spec}(R) \times_X Y \to \mathrm{Spec}(R)$. This is simply the data of a R-point of Y.

4. Quasi-coherent sheaves on stacks

Just as we can define quasi-coherent sheaves on schemes, we can define a quasi-coherent sheaf on a stack as follows:

"Definition" 4.1. For a prestack $\mathfrak{X}$, a quasi-coherent sheaf is one of the following, from concrete to abstract nonsense:

- a quasi-coherent sheaf $\mathcal{F}$ on $\mathfrak{X}$ is the data of, for each ring R and a morphism $f \colon \mathrm{Spec}(R) \to X$, a data of $f^*\mathcal{F} \in \mathrm{QCoh}(\mathrm{Spec}\, R)$, and for any morphism $g \colon \mathrm{Spec}(S) \to \mathrm{Spec}(R)$, a compatible system of isomorphisms

$$g^* f^* \mathcal{F} \simeq (f \circ g)^* \mathcal{F}.$$

- let

$$\mathrm{QCoh}(\mathfrak{X}) := \lim_{\mathrm{Spec}(R)\to\mathfrak{X}} \mathrm{QCoh}(\mathrm{Spec}\, R). \tag{4.1}$$

- QCoh is a functor CRings $\to$ **Ab**, where **Ab** is the category of abelian categories, and we can right Kan extend along the full embedding CRings $\to \mathrm{PreStk}^{\mathrm{op}}$.

The issue is, since pullbacks along arbitrary morphisms $g\colon \mathrm{Spec}(S) \to \mathrm{Spec}(R)$ may not be exact (i.e., $- \otimes_R S$ is not an exact functor on the category of R-modules), the above construction does not quite give us what we want. I believe the definition is fine if $\mathfrak{X}$ is a stack in the fpqc topology, but in any case the better definition is to work with derived categories. Thus we modify (4.1) to

$$\mathcal{D}_{qc}(\mathfrak{X}) := \lim_{\mathrm{Spec}(R)\to\mathfrak{X}} \mathcal{D}_{qc}(\mathrm{Spec}\, R)$$

where $\mathcal{D}_{qc}(\mathrm{Spec}\, R)$ denotes the derived category of R-modules. For this definition to work well, we really need to work with ∞-categories instead of triangulated categories. We ignore all these technicalities below.

Example 4.2. Here are some quasi-coherent sheaves on stacks:

1. any stack $\mathfrak{X}$ has the structure sheaf $\mathcal{O}_{\mathfrak{X}}$. It is defined so that for any morphism $p\colon \mathrm{Spec}(R) \to X$,
$$p^*\mathcal{O}_{\mathfrak{X}} = \mathcal{O}_{\mathrm{Spec}(R)}.$$
2. when $\mathfrak{X} = X^{\wedge}_{Y}$ for a noetherian scheme X and a closed subscheme Y and Y_n denotes the n-th formal neighborhood around Y, we have
$$\mathrm{QCoh}(X^{\wedge}_{Y}) = \lim \mathrm{QCoh}(Y_n).$$
In other words, a quasi-coherent sheaf on $X^{\wedge}_{Y}$ is a compatible system of quasi-coherent sheaves on Y_n.
3. the stack $\mathfrak{X} = B\mathbb{G}_m$ has a tautological line bundle $\mathcal{O}(-1)$, defined so that for any morphism $p\colon \mathrm{Spec}(R) \to \mathfrak{X}$, which corresponds to a line bundle $\mathcal{L}$ on $\mathrm{Spec}(R)$, we let
$$p^*\mathcal{O}(-1) := \mathcal{L}.$$
More generally, given a representation V of an affine group scheme G, there is a corresponding sheaf $\mathcal{F}_V$ on BG, so that for any morphism $p\colon \mathrm{Spec}(R) \to X$ corresponding to a G-bundle $Y \to \mathrm{Spec}(R)$, we can let
$$p^*\mathcal{F}_V := Y \times^G V.$$
In fact, $V \mapsto \mathcal{F}_V$ gives an equivalence $\mathrm{Rep}(G) \simeq \mathrm{QCoh}(BG)$.
4. the scheme $\mathbb{P}^1$ considered as a stack has a tautological line bundle $\mathcal{O}(-1)$, defined so that for any morphism $p\colon \mathrm{Spec}(R) \to \mathbb{P}^1$ corresponding to a sub-line-bundle $L \subset R^2$, we let
$$p^*\mathcal{O}(-1) = L.$$
Then the compatible embeddings $L \subset R^2$ gives rise to an embedding $\mathcal{O}(-1) \hookrightarrow \mathcal{O}^2$ on $\mathbb{P}^1$. Similarly, the stack $\mathbb{A}^1/\mathbb{G}_m$ has a tautological line-bundle $\mathcal{O}(-1)$ with a homomorphism $\mathcal{O}(-1) \to \mathcal{O}$.
5. for a smooth scheme X over a field k of characteristic zero, let X^{dR} be the stack
$$X^{\mathrm{dR}}(R) = X(R/\mathrm{Nil}(R)),$$
where $\mathrm{Nil}(R)$ is the nilpotent radical of R. Then there is an equivalence
$$\text{D-mod}(X) \simeq \mathrm{QCoh}(X^{\mathrm{dR}}).$$
The equivalence also respects cohomology, so the de Rham cohomology of X can be computed as:
$$R\Gamma_{\mathrm{dR}}(X) \simeq R\Gamma(X^{\mathrm{dR}}, \mathcal{O}_{X^{\mathrm{dR}}}).$$

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